Eur. Phys. J. B **8**, 399–404 (1999)

THE EUROPEAN PHYSICAL JOURNAL B

EDP Sciences

Berreman-matrix formulation of light propagation in stratified anisotropic chiral media

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Received 11 May 1998

Abstract. We critically analyze the problem of light propagation in stratified birefringent gyrotropic media by introducing a 4×4 matrix formulation. As a test model we consider the case of short-pitch chiral smectic-C liquid crystals, for which simple descriptions in terms of an effective gyrotropic homogeneous model and of a microscopic periodic structure are available. By comparing reflectances and transmittances from finite and semi-infinite slabs, we show that the homogeneous description is unable to account for some of the properties of the underlying periodic structure, whatever small is the pitch with respect to the light wavelength.

PACS. 78.20.Ek Optical activity – 61.30.-v Liquid crystals

1 Introduction

The rotation of the polarization's direction, per unit thickness, of light propagating in an optically-active material scales as the ratio a/λ^2 between some typical molecular length a and the square of the light wavelength λ. Therefore, optical activity is a clear manifestation of the molecular structure of matter. However, since usually $a \ll \lambda$, it is generally described in terms of macroscopical *homogeneous* constitutive relations [1]. This poses a few problems: to connect the macroscopic quantities appearing in the constitutive relations with the microscopic properties of the medium [2–5], to analyze the internal coherence and the physical implications of the macroscopic equations [6–9], and more generally to determine to what extent a *homogeneous* model can describe a property, such as optical activity, that is entirely due to the *inhomogeneous* nature of the medium.

In the past several homogeneous gyrotropic constitutive relations have been formulated. Born, on the basis of microscopic considerations, proposed the following equations for isotropic media [10]

$$\mathbf{D} = \epsilon \left(\mathbf{E} + \gamma \nabla \times \mathbf{E} \right), \qquad \mathbf{B} = \mu_0 \mathbf{H}. \tag{1.1}$$

In the anisotropic case, they generalize to the Landau form [11]

$$D_i = \epsilon_0 \left(\epsilon'_{ij} E_j + \gamma_{ijk} \frac{\partial E_j}{\partial x_k} \right), \qquad B_i = \mu_0 H_i, \tag{1.2}$$

(here and in the following, summation over repeated indices is implied). In these equations, the contribution to the electric displacement proportional to the spatial derivatives of the electric field describes the spatial inhomogeinity of the medium and, for a plane wave, gives rise to an effective dielectric tensor whose imaginary part linearly depends on the light wavevector components. Other alternative approaches, instead, consider algebraic constitutive equations that mix electric and magnetic fields. An example is given by the Post constitutive relations [12]

$$D_i = \epsilon_0 \epsilon_{ij} E_j + \epsilon_0 c \xi_{ij} B_j, \qquad H_i = \mu_0^{-1} B_i + \epsilon_0 c \zeta_{ij} E_j. \tag{1.3}$$

These latter formulations agree with the microscopic findings that optical activity in chiral molecules is due to the excitation of both electric and magnetic dipoles [2,5]. It can be shown [12] that in the bulk these different approaches are essentially equivalent, but they differ when boundary conditions are considered [13], since the Landau formalism (1.2) does not satisfy energy conservation across boundaries.

In this paper we shall analyze in detail the case of a stratified optically-active medium, giving a solution to the problem in terms of a Berreman-like 4×4 matrix formalism. To develop and test this homogeneous model, we shall specifically consider the case of a chiral smectic-C liquid crystal (S_c^*) [14] having a pitch much smaller than the light wavelength. To a good degree of approximation, a S_c^* can be considered as a locally uniaxial medium, in which the optical axis periodically rotates along a fix direction, describing an helix. The advantage of this system is the availability of simple "microscopic" and macroscopic

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descriptions. In particular, for light propagating *along* the helical axis, exact analytical solutions are available [15] that give a pseudo-rotatory power scaling as $(p/\lambda)^3$, and therefore becoming rapidly negligible as $p/\lambda \to 0$. When the pitch of the helix is small with respect to the light wavelength, instead, an even simpler approximate analytical solution has been recently found, that shows a true rotatory power, scaling as p/λ, for light propagating *orthogonally* to the helical axis [16]. In Section 2, starting from this recently proposed macroscopic model of short-pitch S_c^*'s, we formulate a Berreman-like set of equations for stratified optically-active anisotropic media. In Section 3 we compare reflectances and transmittances from a S_c^* sample computed according to the homogeneous model and to the actual microscopic structure, discussing the limits of validity of the former. Finally, in Section 4 we summarize and discuss our results.

2 Theory

A dielectric periodic medium having a pitch much smaller than the light wavelength can be described in terms of an effective homogeneous model displaying optical activity [16,17]. In particular, for an unbounded chiral smectic-C liquid crystal, the components of the effective dielectric tensor experienced by a plane wave of wavevector $\mathbf{k}$ can be written as [16]

$$(\epsilon_{eff})_{mn} = \epsilon'_{mn} + i\gamma_{mnr}k_r, \tag{2.1}$$

where summation over repeated indices is understood and k_r are the components of $\mathbf{k}$. In the orthogonal Cartesian frame whose z-axis coincides with the helical axis of the S_c^*, the real part of the dielectric tensor is

$$\epsilon' = \begin{pmatrix} \epsilon'_o & 0 & 0 \\ 0 & \epsilon'_o & 0 \\ 0 & 0 & \epsilon'_e \end{pmatrix}, \tag{2.2}$$

with

$$\begin{aligned} \epsilon'_o &= \epsilon_o + \frac{\epsilon_a \epsilon_o \sin^2\alpha}{2(\epsilon_e - \epsilon_a \sin^2\alpha)}, \\ \epsilon'_e &= \epsilon_e - \epsilon_a \sin^2\alpha. \end{aligned} \tag{2.3}$$

Here ϵ_o (resp. ϵ_e) is the local ordinary (resp. extraordinary) dielectric constant of the periodic medium, $\epsilon_a = \epsilon_e - \epsilon_o$ is the dielectric anisotropy, and α is the tilt angle of the S_c^*. The third-rank tensor γ_{mnr} can be expressed as

$$\gamma_{ijk} = k_0^{-1} e_{ijm} g_{mk}, \tag{2.4}$$

where $k_0 = 2\pi/\lambda$ is the vacuum wavevector, e_{ijm} the totally antisymmetric Levi-Civita tensor, and g_{ij} are the components of the second rank gyration tensor [11]

$$\mathrm{g} = g_\perp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2.5}$$

with

$$g_\perp = -\frac{p}{\lambda}\frac{\epsilon_a^2 \sin^2(2\alpha)}{8(\epsilon_e - \epsilon_a \sin^2\alpha)}, \tag{2.6}$$

where p is the S_c^* pitch. The effective dielectric tensor (2.1) has been obtained by considering the propagation of a plane wave in an unbounded periodic medium. It is uniaxial and its imaginary part, which describes the optical activity, displays spatial dispersion, *i.e.*, depends on the light wavevector $\mathbf{k}$.

For non-homogeneous media and arbitrary electromagnetic fields, it might seem natural to generalize the constitutive equation (2.1) according to the Landau formalism (1.2), with the replacement $ik_r \to \partial/\partial x_r$. However, with the Landau constitutive relations, energy conservation requires not only the antisymmetry of the third-rank tensor γ_{ijk}, $\gamma_{ijk} = -\gamma_{jik}$ [11] (which is locally satisfied in a non-dissipative dielectric as the one that we are considering), but also that the sample is homogeneous [18]. This is a signature of the failure of this formulation in the general case. To overcome this problem, we shall recast the constitutive equations in the Post formalism (1.3). Within this formulation, energy is conserved if at any point [1]

$$\epsilon = \epsilon^\dagger, \quad \zeta = -\xi^\dagger. \tag{2.7}$$

As noted by Peterson [12], in a homogeneous sample the constitutive relations (1.2, 1.3, 2.4) are equivalent provided that

$$\begin{aligned} \xi &= i\mathrm{R}, \\ \zeta &= -i\mathrm{R}^t, \\ \epsilon &= \epsilon' + \mathrm{R}\mathrm{R}^t, \\ \mathrm{R} &= \left(\frac{1}{2}\mathrm{Tr}\,\mathrm{g}\right)\mathrm{I} - \mathrm{g}, \end{aligned} \tag{2.8}$$

where Tr indicates the trace and R^t is the transpose of R. More precisely, in the two formulations the physical fields $\mathbf{E}$ and $\mathbf{B}$ are the same, while the fields $\mathbf{D}$ and $\mathbf{H}$ are modified in such a way that Maxwell's equations hold true with the two different constitutive equations. If the medium is not homogeneous, the correspondence (2.8) does not hold any more. In particular, the Post formulation (1.3) ensures energy conservation also in a non-homogeneous sample. For this reason, we shall assume that equations (1.3, 2.8) represent the correct constitutive equations for non-homogeneous gyrotropic materials. Using these equations, for a sample stratified in the z-direction and homogeneous in the transverse directions x and y, we can cast Maxwell's equations in the Berreman form [19]

$$\frac{d\psi}{dz} = ik_0 \mathrm{B}\psi, \tag{2.9}$$

where ψ is the column matrix containing the amplitudes of the transverse fields

$$\psi = \begin{pmatrix} e_x \\ h_y \\ e_y \\ -h_x \end{pmatrix}, \tag{2.10}$$

and the elements of the 4×4 Berreman matrix B are given by

$$\begin{aligned}
B_{11} &= -iR_{12} + (\epsilon_{33} - R_{33}^2)^{-1}(p_0 - iR_{32}) \\
&\quad \times[-\epsilon_{31} + iR_{33}(q_0 - iR_{13})]\,, \\
B_{12} &= 1 - (\epsilon_{33} - R_{33}^2)^{-1}(p_0^2 + R_{32}^2)\,, \\
B_{13} &= -iR_{22} - (\epsilon_{33} - R_{33}^2)^{-1}(p_0 - iR_{32}) \\
&\quad \times[\epsilon_{32} + iR_{33}(p_0 + iR_{23})]\,, \\
B_{14} &= -(\epsilon_{33} - R_{33}^2)^{-1}(p_0 - iR_{32})(q_0 - iR_{31})\,, \\
B_{21} &= \epsilon_{11} + (\epsilon_{33} - R_{33}^2)^{-1}\left[-\epsilon_{31}\epsilon_{13} + iR_{33}\epsilon_{13}(q_0 - iR_{13})\right. \\
&\quad \left.-iR_{33}\epsilon_{31}(q_0 + iR_{13}) - \epsilon_{33}(q_0^2 + R_{13}^2)\right]\,, \\
B_{22} &= iR_{12} - (\epsilon_{33} - R_{33}^2)^{-1}(p_0 + iR_{32}) \\
&\quad \times[\epsilon_{13} + iR_{33}(q_0 + iR_{13})]\,, \\
B_{23} &= \epsilon_{12} + (\epsilon_{33} - R_{33}^2)^{-1}\left[-\epsilon_{13}\epsilon_{32}\right. \\
&\quad +\epsilon_{33}(q_0 + iR_{13})(p_0 + iR_{23}) \\
&\quad \left.-iR_{33}\epsilon_{13}(p_0 + iR_{23}) - iR_{33}\epsilon_{32}(q_0 + iR_{13})\right]\,, \\
B_{24} &= -iR_{11} - (\epsilon_{33} - R_{33}^2)^{-1}(q_0 - iR_{31})[\epsilon_{13} \\
&\quad +iR_{33}(q_0 + iR_{13})]\,, \qquad (2.11)\\
B_{31} &= iR_{11} + (\epsilon_{33} - R_{33}^2)^{-1}(q_0 + iR_{31})[-\epsilon_{31} \\
&\quad +iR_{33}(q_0 - iR_{13})]\,, \\
B_{32} &= -(\epsilon_{33} - R_{33}^2)^{-1}(q_0 + iR_{31})(p_0 + iR_{32})\,, \\
B_{33} &= iR_{21} - (\epsilon_{33} - R_{33}^2)^{-1}(q_0 + iR_{31})[\epsilon_{32} \\
&\quad +iR_{33}(p_0 + iR_{23})]\,, \\
B_{34} &= 1 - (\epsilon_{33} - R_{33}^2)^{-1}(q_0^2 + R_{31}^2)\,, \\
B_{41} &= \epsilon_{21} + (\epsilon_{33} - R_{33}^2)^{-1}\left[-\epsilon_{23}\epsilon_{31} + iR_{33}\epsilon_{23}(q_0 - iR_{13})\right. \\
&\quad \left.+iR_{33}\epsilon_{31}(p_0 - iR_{23}) + \epsilon_{33}(q_0 - iR_{13})(p_0 - iR_{23})\right]\,, \\
B_{42} &= iR_{22} + (\epsilon_{33} - R_{33}^2)^{-1}(p_0 + iR_{32})[-\epsilon_{23} \\
&\quad +iR_{33}(p_0 - iR_{23})]\,, \\
B_{43} &= \epsilon_{22} + (\epsilon_{33} - R_{33}^2)^{-1}\left[-\epsilon_{32}\epsilon_{23} - \epsilon_{33}(p_0^2 + R_{23}^2)\right. \\
&\quad \left.-iR_{33}\epsilon_{23}(p_0 + iR_{23}) + iR_{33}\epsilon_{32}(p_0 - iR_{23})\right]\,, \\
B_{44} &= -iR_{21} - (\epsilon_{33} - R_{33}^2)^{-1}(q_0 - iR_{31})[\epsilon_{23} \\
&\quad -iR_{33}(p_0 - iR_{23})]\,.
\end{aligned}$$

Equations (2.9, 2.11) allow to solve the general problem of light propagation in an arbitrarily stratified anisotropic chiral medium. For the special case of discrete isotropic chiral layers, a somewhat different 4×4 transfer matrix approach has been developed a few years ago [20]. The longitudinal components of the electromagnetic field can be expressed as a function of the transverse ones

$$\begin{aligned}
e_z &= (\epsilon_{33} - R_{33}^2)^{-1}\left\{[-\epsilon_{31} + iR_{33}(q_0 - iR_{13})]e_x\right. \\
&\quad -(p_0 + iR_{32})h_y - [\epsilon_{32} + iR_{33}(p_0 + iR_{23})]e_y \\
&\quad \left.+(q_0 - iR_{31})h_x\right\}\,, \\
h_z &= (\epsilon_{33} - R_{33}^2)^{-1}\left\{-[iR_{33}\epsilon_{31} + \epsilon_{33}(q_0 - iR_{13})]e_x\right. \\
&\quad -iR_{33}(p_0 + iR_{32})h_y \\
&\quad +[-iR_{33}\epsilon_{32} + \epsilon_{33}(p_0 + iR_{23})]e_y \\
&\quad \left.+iR_{33}(q_0 - iR_{31})h_x\right\}\,. \qquad (2.12)
\end{aligned}$$

As in the case of non gyrotropic media [21], we can define a scalar product between two Berreman vectors ψ_a, ψ_b

$$(\psi_a, \psi_b) \equiv \psi_a^\dagger \mathrm{M} \psi_b\,, \qquad (2.13)$$

where M is the metric matrix

$$\mathrm{M} = \begin{pmatrix} 0&1&0&0 \\ 1&0&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{pmatrix}. \qquad (2.14)$$

It can be checked that for lossless media, with the definition (2.11), the eigenvectors of the Berreman matrix are orthogonal with respect to the scalar product (2.13). This property ensures energy conservation through boundaries and non-homogeneous samples [21].

3 Comparison with the exact model

An exact numerical approach to compute the optical properties of periodic dielectric stratified media, based on a Bloch-wave decomposition of the electromagnetic field, has been proposed some time ago [22] and applied to chiral liquid crystals in phase grating geometries [23] and with pitches smaller than the light wavelength [17]. In these works it has been considered only the case in which the periodicity is one-dimensional; the extension to two and three-dimensional situations is trivial, but the computing complexity increases dramatically. Moreover, if the medium is not strictly periodic, or is made of slabs having different periodicities, the application of this approach becomes clumsy. For these reasons, the homogeneous models of short-pitch dielectric media are of particular interest. It must also be noted that the optical properties of solid crystals or chiral organic molecules are customarily described only in terms of constitutive relations of the kind of equations (1.2) or (1.3), since in this case the spatial variations of the optical responses are on atomic scales. In the past, different inequivalent models have been employed to compute reflectances and transmittances through gyrotropic media [6,7]. Here, by analyzing the specific case of a S_c^* liquid crystal, for which both a homogeneous and a "microscopic" model is available, we want to test the validity of such homogeneous descriptions against the exact properties of the true periodic structure.

We consider a S_c^* liquid crystal slab having its helical axis parallel to the x-axis of a Cartesian coordinate system whose z-axis coincides with the direction of stratification. By neglecting any biaxiality and intrinsic molecular chirality, the local optical tensor of the S_c^* can be written as

$$\epsilon = \epsilon_0\left(\epsilon_o \mathrm{I} + \epsilon_a \mathbf{n} \otimes \mathbf{n}\right), \qquad (3.1)$$

where I is the unit tensor and $\mathbf{n}$ is the nematic director, that uniformly rotates around the helical axis x describing a cone of semi-aperture α

$$\mathbf{n} = \cos\alpha\hat{\mathbf{x}} + \sin\alpha\cos(qx)\hat{\mathbf{y}} + \sin\alpha\sin(qx)\hat{\mathbf{z}}, \qquad (3.2)$$

where $q = 2\pi/p$ is the S_c^* wavevector and $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are the unit coordinate vectors. The details of the calculation of light propagation in such kind of periodic media

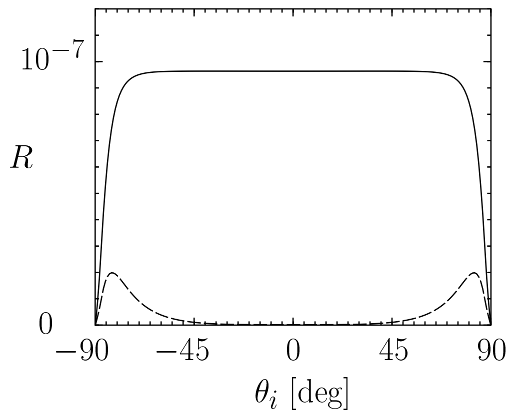

Fig. 1. TE-TM power reflection coefficient R as a function of the polar incidence angle θ_i for a semi-infinite slab. The azimuthal incidence angle is $\phi_i = 0°$. Here and in the following figures $n_i = 1.5$, $n_o \equiv \epsilon_o^{1/2} = 1.5$, $n_e \equiv \epsilon_e^{1/2} = 1.7$, and $\alpha = 45°$. Continuous line: homogeneous model; dashed: periodic exact model. The pitch of the S_c^* is $p = 0.1\,\lambda$.

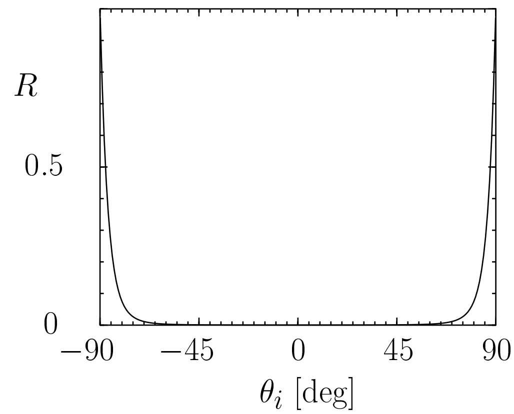

Fig. 2. Same as Figure 1, but for the TE-TE geometry. On this scale the two curves are indistinguishable.

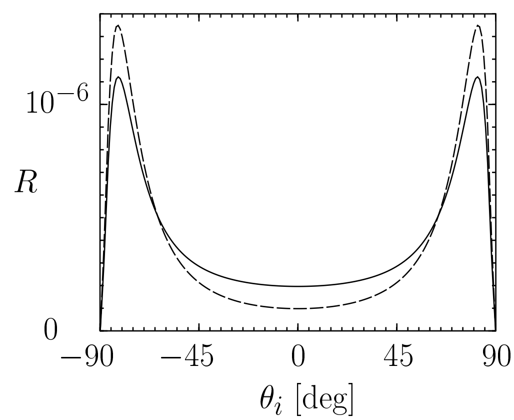

Fig. 3. Same as Figure 1, but for azimuthal incidence angle $\phi_i = 1°$.

in terms of Bloch-waves have already been presented elsewhere [23,17] and will not be discussed here.

To avoid interference effects, we start by considering a single interface, orthogonal to the z-axis, between the S_c^* and a homogeneous isotropic dielectric medium having index of refraction n_i. Figure 1 shows a typical depolarized intensity reflection curve for incident transverse electric (TE) and reflected transverse magnetic (TM) waves in the isotropic medium. In this figure, the incidence plane coincides with the (x, z)-plane ($\phi_i = 0$). This geometry ensures that an incident TE (resp. TM) polarization corresponds in the homogeneous model to an ordinary (resp. extraordinary) ray with respect to the real part of the dielectric tensor of the gyrotropic medium. Therefore, a non-zero depolarized reflectance is a signature of gyrotropy. In Figure 1 the dashed line is the depolarized reflectance numerically computed according to the exact periodic model. The solid line is the corresponding curve obtained by a standard Berreman approach [21] using our Berreman matrix for gyrotropic media (2.11). As it is apparent, the depolarized reflectivity computed according to the homogeneous model is much higher than that given by the true periodic model. For instance, with the parameters used in Figure 1, and in particular for a S_c^* pitch $p = 0.1\,\lambda$, the exact calculation gives an extremely small depolarized power reflection coefficient at normal incidence of 1.6×10^{-10}, while the homogeneous model gives a 600 times larger value of 9.6×10^{-8}. This huge relative error remains practically unchanged as $p/\lambda \to 0$, even if, of course, the absolute error goes to zero, since the depolarized power reflection scales as $(p/\lambda)^2$. Such a behavior is a signature of an incorrect treatment, in the homogeneous model, of the effects of a surface. When the periodicity is taken into account, the surface generates an evanescent wave extending on a thickness of the order of p, for $p \ll \lambda$. The coupling between the evanescent and the propagating waves effectively smooths the interface, reducing the depolarized reflection by a quantity of the order of p/λ with respect to the value given by the homogeneous model, which neglects the presence of such an evanescent wave. It must be noticed, however, that the actual values of these discrepancies are so tiny that it is questionable whether they can be observed in a true experimental situation.

When simply measurable quantities are considered, as the non-depolarized TE-TE reflectance shown in Figure 2, the two models give practically the same answer: in the case shown in Figure 2 the maximum relative error between the true and the homogeneous model is less than 10^{-2}.

As soon as we tilt the incidence plane away from the helical axis, which coincides with the optical axis of the homogeneous model, the contribution of the birefringence to the depolarized reflections dominates, giving increasingly larger depolarized reflectances. Correspondingly, the differences between the homogeneous model and the exact calculation rapidly disappear, as shown in Figure 3. This makes even more delicate an experimental verification of these theoretical results.

Let us now see what happens when, instead of a semi-infinite medium, one considers a finite slab sandwiched between two identical isotropic dielectrics. In this case, interference effects due to multiple reflections are expected, together with an interplay between the gyrotropy and the

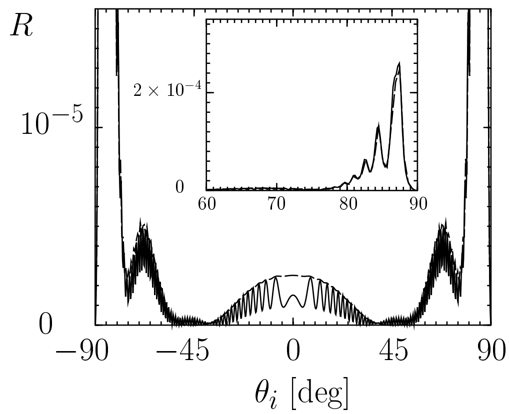

Fig. 4. TE-TM reflection coefficient for a finite slab with thickness $d = 25\,\lambda$ and pitch $p = 0.1\,\lambda$. The azimuthal incidence angle is $\phi_i = 0°$. The inset shows the grazing incidence peaks. Solid line: homogeneous model; dashed line: exact periodic model.

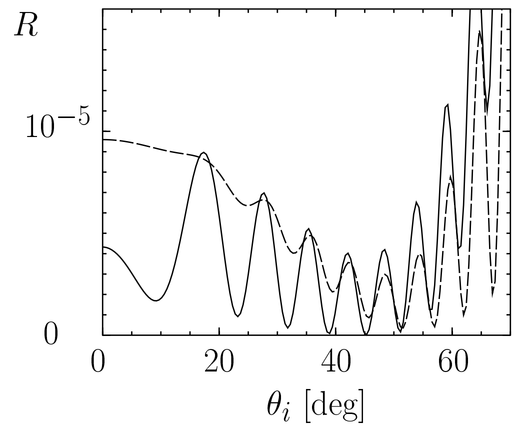

Fig. 5. Same as Figure 4 but for $p = 0.3\,\lambda$ and $d = 5\,\lambda$.

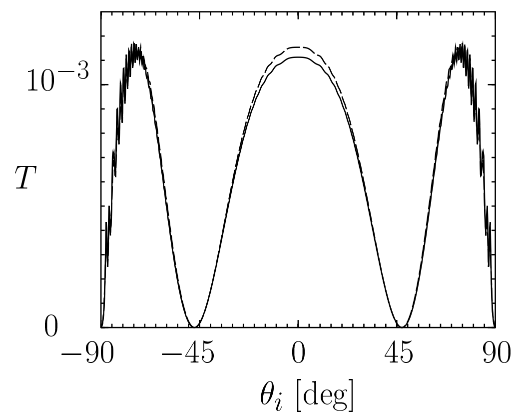

Fig. 6. TE-TM power transmission coefficient T corresponding to Figure 4.

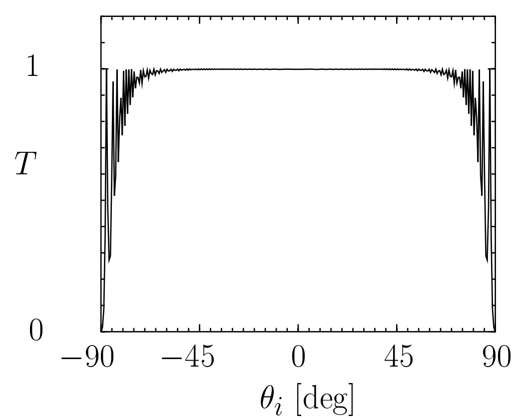

Fig. 7. TE-TE power transmission coefficient T corresponding to Figure 4.

birefringence. This is indeed shown in Figure 4, which gives the depolarized TE-TM reflection coefficient for a thin sample with thickness $d = 25\,\lambda$. Two systems of fringes are present. The large ones are mainly due to the interference between the ordinary (TE) and the extraordinary (TM) rays propagating inside the birefringent sample: they are almost the same in both the models. The shallow fine fringes, on the other hand, are due to the interference of the extraordinary (TM) ray with itself in the multiple reflections. They are almost absent in the true periodic model because of the very low depolarized reflection at the first interface. This is confirmed by Figure 5: increasing the pitch to $p = 0.3\,\lambda$ enhances the depolarized reflectivity of the first interface, in such a way that the periodic structure now displays marked fringes comparable with those given by the homogeneous model. Here again, however, an experimental verification is rather difficult, due to both the relatively small value of the depolarized reflectivity and to smoothing effect caused, *e.g.*, by the finite angular aperture of the light beam. The relative errors in the depolarized transmittances are much smaller, as shown in Figure 6, and practically unobservable in the non-depolarized case, as shown for the TE-TE geometry in Figure 7.

We can therefore conclude that the homogeneous models of optical activity correctly describe the bulk behavior, such as, *e.g.*, the optical rotation [16,17], but seem intrinsically unable to account for some of the details of the boundary effects, whatever small is the periodicity. Another situation in which the homogeneous models trivially fail is when the pitch becomes comparable to the light wavelength, in such a way that Bragg reflections occur. This can happen before that any diffracted propagating beam appears outside of the periodic medium. This case is illustrated in Figure 8 for the TM-TM reflectance. Here the pitch of the S_c^* is $p = 0.33\,\lambda$ and, with respect to the previous cases, the helical axis is anti-clockwise rotated of 20° in the (x, z)-plane with respect to the x-axis.

Finally, we note that a degenerate case in which the homogeneous models never hold is when the boundary planes are exactly orthogonal to the direction of the periodicity [24]: this fact can be understood either by recognizing that the orientation of the local optical axis on the boundary planes – orientation that does not enter into the homogeneous models – plays a determinant role, or, equivalently, by considering that as the direction of periodicity

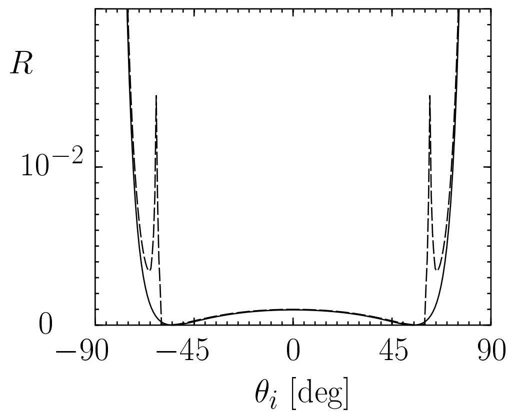

Fig. 8. TM-TM reflectance for a semi-infinite slab with $p = 0.33\,\lambda$ and S_c^* helical axis inclined of 20° with respect to the boundary planes. The azimuthal incidence angle is $\phi_i = 0°$. Solid line: homogeneous model; dashed line: exact periodic model.

becomes orthogonal to the boundary planes, the period of the structure projected onto these planes diverges, giving rise to propagating Bragg peaks.

4 Conclusions

Optical activity is one of the most striking manifestations of the molecular structure of matter: the rate of rotation of the plane of polarization of a light beam passing through an optically-active medium is proportional to a/λ^2, where a is a typical molecular length. However, since usually $a \ll \lambda$, it is generally understood that optical activity can be described in terms of homogeneous *continuum* models.

In this paper, starting from the effective dielectric tensor experienced by a plane wave traveling in an infinite periodic dielectric medium having a pitch much smaller than the light wavelength, we have developed a 4×4 matrix evolution equation for stratified gyrotropic media. To preserve energy conservation, we have recasted the constitutive equations in the Post formalism. Such a matrix formulation, which extends the usual Berreman equations to gyrotropic media, allows to easily compute the optical properties of one-dimensional stratified media. As a test model, we have considered the case of a short pitch chiral smectic-C liquid crystal. The interest in such a system stems from the availability of both a local "microscopic" model, in terms of a periodic dielectric structure, and of a homogeneous description, valid for pitches short with respect to the light wavelength. The homogeneous model corresponds to an optically-active medium for light propagating orthogonally to the helix axis [24]. By considering a finite or semi-infinite S_c^* slab surrounded by a dielectric isotropic medium, we have compared reflectances and transmittances for linearly polarized incident light, computed both taking into account of the full periodic structure [22] and according to the homogeneous model [11,12]. We have found that the depolarized reflection coefficients computed on the basis of the true periodic structure differ from the homogeneous predictions by some orders of magnitude, with a relative error that remains constant as the pitch is decreased. Therefore, in principle, such a deviation should persist even in a solid crystal, in which the pitch is on atomic scales. However, in this case the absolute value of the depolarized reflections are so tiny that they are hardly measurable. The reduction of the depolarized reflection coefficients causes a strong suppression of interference fringes in thin samples.

In conclusion, the homogeneous models of optically-active media correctly describe the bulk behavior but are intrinsically unable to give some of the details of the boundary effects. Another trivial situation in which they fail is when the periodicity becomes comparable to the light wavelength, such that Bragg reflections occur, even before that any propagating diffracted beam appears outside of the periodic medium. Finally, the homogeneous models completely fail when the direction of periodicity is orthogonal to the boundaries [16,24], independently of the pitch value. This can be understood either because the period projected onto the boundary planes diverges, or, equivalently, because in this geometry the local initial and final phases of the periodic structure critically determine their optical properties.

We thank Prof. C. Oldano for fruitful discussions.

References

1. Jin Au Kong, *Electromagnetic wave theory* (John Wiley & Sons, New York, 1990).
2. M. Born, Phys. Z. **16**, 251 (1915).
3. C.W. Oseen, Ann. Phys. **48**, 1 (1915).
4. F. Gray, Phys. Rev. **7**, 472 (1916).
5. E.U. Condon, Rev. Mod. Phys. **9**, 432 (1937).
6. M.P. Silverman, Lett. Nuovo Cimento **43**, 378 (1985).
7. M.P. Silverman, J. Opt. Soc. Am. A **3**, 830 (1986).
8. A. Lakhtakia, V.V. Varadan, V.K. Varadan, J. Opt. Soc. Am. A **5**, 175 (1988).
9. S. Bassiri, C.H. Papas, N. Engheta, J. Opt. Soc. Am. A **5**, 1450 (1988).
10. M. Born, *Optik* (Springer-Verlag, Heidelberg, 1972).
11. L. Landau, E. Lifchitz, *Électrodynamique des milieux continus* (Mir, Moscou, 1969).
12. R.M. Peterson, Am. J. Phys. **43**, 969 (1975).
13. R.M. Hornreich, S. Shtrikman, Phys. Rev. **171**, 1065 (1968).
14. P.G. de Gennes, J. Prost, *The Physics of Liquid Crystals* (Clarendon Press, Oxford, 1993).
15. H. de Vries, Acta Crystallogr. **4**, 219 (1951).
16. C. Oldano, M. Rajteri, Phys. Rev. B **54**, 10273 (1996).
17. P. Galatola, Phys. Rev. E **55**, 4338 (1997).
18. F.I. Fedorov, Opt. Spectrosc. (USSR) **6**, 49 (1959).
19. D.W. Berreman, J. Opt. Soc. Am. **62**, 502 (1972).
20. D.L. Jaggard, X. Sun, J. Opt. Soc. Am. A **9**, 804 (1992).
21. C. Oldano, Phys. Rev. A **40**, 6014 (1989).
22. K. Rokushima, J. Yamakita, J. Opt. Soc. Am. A **73**, 901 (1983).
23. P. Galatola, C. Oldano, P.B. Sunil Kumar, J. Opt. Soc. Am. A **11**, 1332 (1994).
24. P. Hubert, P. Jägemalm, C. Oldano, M. Rajteri, Phys. Rev. E **58**, 3264 (1998).